

Title: Combinatorial Laplacians and high dimensional tree numbers

Abstract: Combinatorial Laplacians provide important enumeration methods in topological combinatorics. For a finite chain complex $\{C_i, \partial_i\}$, combinatorial Laplacians Δ_i on C_i are defined by

$$\Delta_i = \partial_{i+1} \partial_{i+1}^t + \partial_i^t \partial_i.$$

We will review applications of Δ_0 in computing the tree numbers for graphs and in solving discrete Laplace equations for networks. In general, the boundary operators ∂_i are used to define high-dimensional trees as a generalization of spanning trees for graphs. We will demonstrate an intriguing relation between high-dimensional tree numbers and $\det \Delta_i$ for acyclic complexes, based on combinatorial Hodge theory. If time permits, an important role of combinatorial Laplacians in topological data analysis will be discussed.